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# An analysis of errors caused by leakage currents and unintentional potential groundings in the electrical resistivity method

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ABSTRACT

A common error in the electrical resistivity method occurs when a cable connecting to a current or potential electrode is inadvertently grounded at a point other than the intended electrode, thus creating an extra electrode. In this paper, we derive expressions for the magnitude of the induced error of the inadvertent electrode as a function of the position for a homogeneous infinite half-space for a general four-electrode array. We also derive a general expression for the magnitude of the error in terms of the current densities for ground with a general resistivity distribution. We compare the theoretical results for homogeneous ground with small-scale field surveys and find them to be in good agreement. We show that the error in the measured apparent resistivity has a value that is proportional to the apparent resistivity for an array made up of the inadvertent electrode and the electrode to which it is connected. We find that the induced error becomes very large if the additional electrode is a current electrode and it is placed near a potential electrode or if it is a potential electrode and it is placed near a current electrode. The magnitude of the error is the same whether the additional electrode is a current electrode or a potential electrode. The induced error approaches a constant non-zero value as the additional electrode is moved towards infinity. Resistivity heterogeneity in the ground can either increase or decrease the error depending on the sensitivity of the array formed by the inadvertent electrode and the electrode to which it is connected.



## 1. Introduction

The electrical resistivity method is one of the oldest geophysical methods, but it remains widely used (Reynolds, 2011). In recent years, advances in computation for inversion have made 2D surveys routine and 3D and 4D surveys more common (see Loke et al. (2013) for a review). A source of error in the electrical resistivity method involves the unintentional grounding of one of the cables connected to the current or potential electrodes. This grounding can occur, for example, if the insulation on the cable becomes damaged or if wet conditions form a continuous film of moisture along the cable to the connection point with the current source or voltmeter. Metal cable reels can often be points of unintentional grounding (Zohdy, 1968). In multinode systems where nodes are connected via switches to current and potential cables, unintentional grounding may occur if a switch malfunctions and a node is unintentionally active.

To date, the only paper dealing with the effects of leakage currents and unintentional groundings is that of Zohdy (1968), in which he used formulas derived by Dakhnov (1953). Zohdy (1968) presented a formula for the magnitude of the error caused by a leakage current for the special case of a Schlumberger array in ground with constant resistivity. He also discussed characteristic effects of leakage currents on vertical electric sounding curves for Schlumberger arrays. Practical advice given for determining the presence of a current leakage was to disconnect each of the current electrodes in turn and observe whether there remained a measurable potential difference between the potential electrodes when one current electrode was disconnected. Although Zohdy (1968) also gave formulas for the error caused by unintentionally grounded potential cables, he indicated that their effects were likely to be small for the case of a Schlumberger array because of the location of the potential electrodes near the center of the array and at a significant distance from the current electrodes. However, we show that, for other array types, the effects of unintentionally grounded potential cables are likely to be as significant as those due to current-carrying cables.

The effects of an unintentionally grounded cable can be modeled as an additional electrode connected to one of the other electrodes in the array. The importance of the additional electrode and the magnitude of the induced error decrease with the magnitude of the contact resistance of the additional electrode with the ground. In the following sections, we derive expressions for the error induced in a resistivity survey arising due to an unintentional additional electrode in an infinite half-space with constant resistivity when the extra electrode is both a

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current electrode and a potential electrode. Additionally, we show that these expressions obey the reciprocity theorem for resistivity. We then derive expressions for the sensitivity to resistivity heterogeneity in the ground from the error caused by additional electrodes. Subsequently, we compare the predictions of the formulas for the error in constant resistivity ground to the results of a small-scale field survey and show that they are in excellent agreement. Finally, we examine the effects of unintentional grounding points for a model with constant resistivity and examine the effects of resistivity heterogeneity on the magnitude of the error for a simple dyke model.

## 2. Theory

### 2.1. Homogeneous half-space — current cable connected to ground

We consider the situation shown in Fig. 1a. Current electrodes are $A$ and $B$, and there is a grounding in the wire connecting the current source to electrode $A$, creating electrode $C$. Because the potentials in electrodes $A$ and $C$ are the same, the current flowing through them is $(1-\alpha)$ and $\alpha$, respectively assuming that the contact resistances are much greater than the resistance through the ground. Here, $I$ is the total current and $\alpha = R_A/(R_A+R_C)$, where $R_A$ and $R_C$ are the contact resistances of electrodes $A$ and $C$. If there is no unintentional grounding, $R_C \gg R_A$ and $\alpha \to 0$. Although we have drawn a Schlumberger array geometry for the diagram, the following analysis is general to any location of the five electrodes when electrode $C$ is connected to electrode $A$.

The potential at an electrode $M$ (with position $\mathbf{r}_M$) is the sum of the potentials due to the current electrodes at $A$, $B$, and $C$. Using the formula for a point source in an infinite half-space of resistivity $\rho$ (e.g., Telford et al., 1990) gives

$$V(\mathbf{r}_M) = \frac{I\rho}{2\pi}\left(\frac{1-\alpha}{r_{AM}} + \frac{\alpha}{r_{CM}} - \frac{1}{r_{BM}}\right), \tag{1}$$

where $r_{AM}$ indicates the distance between electrodes $A$ and $M$; other distances are defined similarly.

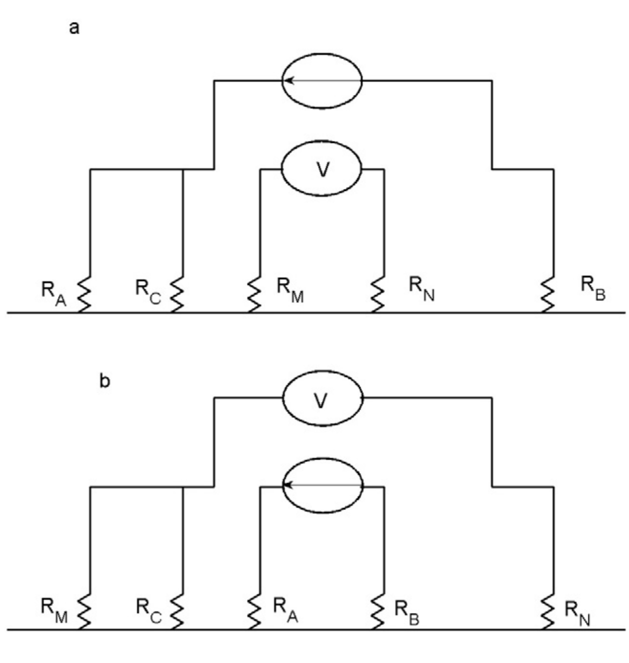


**Fig. 1.** a) Resistivity array with current leakage on a cable connecting to electrode $A$ resulting in an additional electrode $C$. b) Resistivity array with cable connecting to potential electrode $M$ grounded creating an additional potential electrode $C$. Note that the arrays in a and b are the reciprocals of one another with the substitutions $A \leftrightarrow M$ and $B \leftrightarrow N$.

The potential at electrode $N$ (with position $\mathbf{r}_N$) can be found in the same way so that the potential difference between electrodes $M$ and $N$ can be written

$$V(\mathbf{r}_M) - V(\mathbf{r}_N) = \frac{I\rho}{2\pi}\left(\frac{1-\alpha}{r_{AM}} - \frac{1-\alpha}{r_{AN}} + \frac{\alpha}{r_{CM}} - \frac{\alpha}{r_{CN}} - \frac{1}{r_{BM}} + \frac{1}{r_{BN}}\right). \tag{2}$$

The resulting apparent resistivity, $\rho_{ae}$, which is in error due to the extra electrode, is then

$$\rho_{ae} = \frac{k_g\rho}{2\pi}\left(\frac{1-\alpha}{r_{AM}} - \frac{1-\alpha}{r_{AN}} + \frac{\alpha}{r_{CM}} - \frac{\alpha}{r_{CN}} - \frac{1}{r_{BM}} + \frac{1}{r_{BN}}\right). \tag{3}$$

Here, $k_g$ is the geometrical factor for the four-electrode array without electrode $C$. We can calculate the relative error as $\delta\rho = (\rho_{ae} - \rho_a)/\rho_a$, where $\rho_a$ is the apparent resistivity without the additional electrode. Because

$$\rho_a = \frac{k_g\rho}{2\pi}\left(\frac{1}{r_{AM}} - \frac{1}{r_{BM}} - \frac{1}{r_{AN}} + \frac{1}{r_{BN}}\right), \tag{4}$$

we can see that

$$\rho_{ae} - \rho_a = \alpha\frac{k_g\rho}{2\pi}\left(-\frac{1}{r_{AM}} + \frac{1}{r_{AN}} + \frac{1}{r_{CM}} - \frac{1}{r_{CN}}\right). \tag{5}$$

So, the error is proportional to the apparent resistivity for an array with current electrodes at $A$ and $C$ with current $\alpha I$.

The relative error is then

$$\delta\rho = \alpha\frac{-r_{AM}^{-1} + r_{AN}^{-1} + r_{CM}^{-1} - r_{CN}^{-1}}{r_{AM}^{-1} - r_{AN}^{-1} - r_{BM}^{-1} + r_{BN}^{-1}}, \tag{6}$$

which is linearly proportional to the magnitude of the current leakage. An equivalent form of Eq. (6) is

$$\delta\rho = \frac{\alpha k_g}{2\pi}\left(-r_{AM}^{-1} + r_{AN}^{-1} + r_{CM}^{-1} - r_{CN}^{-1}\right). \tag{7}$$

For the special case of a Schlumberger array, Eq. (7) can be shown to be

$$\delta\rho = \alpha\left[\frac{k_g}{2\pi}\left(r_{CM}^{-1} - r_{CN}^{-1}\right) - \frac{1}{2}\right], \tag{8}$$

which is the form reported by Zohdy (1968) based on the work of Dakhnov (1953).

### 2.2. Homogeneous half-space — potential cable connected to ground

We consider the array shown in Fig. 1b that represents the reciprocal of the array shown in Fig. 1a with the exchanges $A \leftrightarrow M$ and $B \leftrightarrow N$ in order to maintain the convention that $A$ and $B$ are current electrodes and $M$ and $N$ are potential electrodes. The reciprocity theorem for resistivity indicates that the ratio of the voltage measured between $M$ and $N$ to the current injected at $A$ and $B$ is identical for the reciprocal arrays. The reciprocity theorem holds for electrodes of any geometry, and so it must hold even when $A$ or $M$ is connected to an additional electrode $C$ at another location (e.g., Parasnis, 1988). As a result, the error in the measurement caused by an additional electrode connected to one of the potential electrodes must be of exactly the same form as Eq. (6). We show this independently below.

When the cable between the voltmeter and $M$ is connected to ground, an additional electrode $C$ is created. Take $R_C$ to be the contact resistance at electrode $C$ and $R_M$ to be the contact resistance at electrode $M$. Assuming that the contact resistances are much larger than the resistance through the ground, the potential in the ground at positions $\mathbf{r}_C$

and $\mathbf{r}_M$ is not affected by the short circuit between electrodes $M$ and $C$.

Here, we must distinguish between the potential $V(\mathbf{r}_M)$ in the ground at the position of electrode $M$ and the potential $Ve(\mathbf{r}_M)$ in the electrode. These are slightly different because electrode $M$ is connected to electrode $C$, and it is separated from the ground by contact resistance $R_M$. Some small amount of current must run through the potential electrodes in order to make a voltage measurement. Assuming that $R_C$ and $R_M$ are much less than the impedance of the voltmeter, $Ve(\mathbf{r}_M)$ is given by

$$Ve(\mathbf{r}_M) = \frac{R_C V(\mathbf{r}_M) + R_M V(\mathbf{r}_C)}{R_C + R_M} = (1-\alpha)V(\mathbf{r}_M) + \alpha V(\mathbf{r}_C), \quad (9)$$

with $\alpha = R_M/(R_M + R_C)$. If the cable is everywhere well insulated, $R_C \gg R_M$ or $\alpha \ll 1$, then $V_e(\mathbf{r}_M) \approx V(\mathbf{r}_M)$.

If there is no grounding in the cable connecting to electrode $N$, then $Ve(\mathbf{r}_N) = V(\mathbf{r}_N)$. The potentials $V(\mathbf{r}_N)$, $V(\mathbf{r}_C)$, and $V(\mathbf{r}_M)$ can be calculated from the formulas for current injection at electrodes $A$ and $B$ for an infinite half-space if the ground is assumed to be homogeneous. This gives for the potential difference between electrodes $M$ and $N$

$$Ve(\mathbf{r}_M) - Ve(\mathbf{r}_N) = \frac{I\rho}{2\pi}\left[(1-\alpha)r_{AM}^{-1} - (1-\alpha)r_{BM}^{-1} + \alpha r_{AC}^{-1} - \alpha r_{BC}^{-1} - r_{AN}^{-1} + r_{BN}^{-1}\right]. \quad (10)$$

Note that Eq. (10) is identical to Eq. (2) with the exchange of indices $A \leftrightarrow M$ and $B \leftrightarrow N$, as they must be because of reciprocity.

In this case, the relative error in the apparent resistivity is given by

$$\delta\rho = \alpha \frac{-r_{AM}^{-1} + r_{BM}^{-1} + r_{AC}^{-1} - r_{BC}^{-1}}{r_{AM}^{-1} - r_{BM}^{-1} - r_{AN}^{-1} + r_{BN}^{-1}}. \quad (11)$$

Eq. (11) is again the same as Eq. (6) with the exchange of indices $A \leftrightarrow M$ and $B \leftrightarrow N$. For the special case of a Schlumberger array, Eq. (11) can be written as

$$\delta\rho = \alpha\left[\frac{k_g}{2\pi}\left(r_{AC}^{-1} - r_{BC}^{-1}\right) - \frac{1}{2}\right], \quad (12)$$

which was the form reported by Zohdy (1968).

Examining Eqs. (6) and (11), we can see that the relative error approaches $\pm\infty$ if $r_{CM}$ or $r_{CN}$ approach zero for the case of the leaking current or if $r_{AC}$ or $r_{BC}$ approach zero for the case of the grounded potential wire. In other words, if the unintentionally grounded cable is current carrying, the error is very large if it is close to one of the potential electrodes whereas if the unintentionally grounded cable is connected to a potential electrode, the error is very large if it is located close to one of the current electrodes. Zohdy (1968) noted this effect for the case of a Schlumberger array with a leakage current.

We can also see that the error vanishes if the extra grounding occurs at the same location as the electrode to which it is connected because we can set the index $C$ to be $A$ in Eq. (6) and $C$ to be $M$ in Eq. (11) under these conditions. This indicates that if there are suspected sources of current leakage such as cable reels, it is best to locate these close to the electrodes to which they are attached, if possible. As we have shown in a subsequent section, the electrode to which the extra grounding point is connected always lies on a 0 contour when the error is plotted as a function of the position of the additional electrode.

Eqs. (6) and (11) also indicate that the error in the apparent resistivity does not go to 0 if the extra electrode is moved far away from the array except when the extra grounding is connected to the infinite electrode of a pole–pole or pole–dipole array. If electrode $C$ is taken to infinity, then we have

$$\delta\rho(\infty) = \alpha \frac{-r_{AM}^{-1} + r_{AN}^{-1}}{r_{AM}^{-1} - r_{AN}^{-1} - r_{BM}^{-1} + r_{BN}^{-1}} \quad (13)$$

for the grounded current wire, whereas for the extra potential electrode, we have

$$\delta\rho(\infty) = \alpha \frac{-r_{AM}^{-1} + r_{BM}^{-1}}{r_{AM}^{-1} - r_{AN}^{-1} - r_{BM}^{-1} + r_{BN}^{-1}}. \quad (14)$$

For the special case of Schlumberger arrays, we get $\delta\rho = -\alpha/2$ for both cases (as can also be easily seen from Eq. (8)). For the case of the extra current electrode, the error does not go to zero because even though electrode $C$ is too far away for its potential to affect the measurement array, it is drawing $\alpha I$ current away from the intended electrode $A$ and so changing the potential created by $A$ in the array.

For the case of the extra potential electrode at infinity, although it is too far away to be affected by the potential of the current injection, it is still biasing the potential of electrode $M$ towards 0 and hence creating an error.

### 2.3. Sensitivities with a general resistivity distribution

We derive the expression for the sensitivity of the resistivity method with no extra grounding points and then compare it with the form for the sensitivity with extra grounding points.

Consider an array with four electrodes consisting of current electrodes $A$ and $B$ and potential electrodes $M$ and $N$. If we approximate the electrodes as points, the governing equation for conservation of charge due to current injection at $A$ and $B$ is

$$\nabla \cdot \mathbf{J}_{AB} = I_{AB}[\delta(\mathbf{r} - \mathbf{r}_A) - \delta(\mathbf{r} - \mathbf{r}_B)], \quad (15)$$

(Geselowitz, 1971). Here $\mathbf{J}_{AB}$ is the current density field, $I_{AB}$ is the total current, $\delta$ represents the Dirac delta function, and $\mathbf{r}_A$ and $\mathbf{r}_B$ represent the positions of electrodes $A$ and $B$. The electrical potential produced by these currents is $V_{AB}$, and Ohm's law gives $\mathbf{J}_{AB} = -\nabla V_{AB}/\rho$.

We note that the system is linear, so we can define $\mathbf{J}_{AB} = \mathbf{J}_A - \mathbf{J}_B$, where $\mathbf{J}_A$ and $\mathbf{J}_B$ are current densities produced by positive current injection at locations $\mathbf{r}_A$ and $\mathbf{r}_B$, respectively. We can further define normalized current densities, $\mathbf{j}_A = \mathbf{J}_A/I_{AB}$ and $\mathbf{j}_B = \mathbf{J}_B/I_{AB}$, and $\mathbf{j}_{AB} = \mathbf{j}_A - \mathbf{j}_B$.

Using Eq. (15), we can then write

$$\nabla \cdot \mathbf{j}_{AB} = \delta(\mathbf{r} - \mathbf{r_A}) - \delta(\mathbf{r} - \mathbf{r}_B). \quad (16)$$

We next consider the reciprocal array, where current is injected between electrodes $M$ and $N$ to produce a potential $V_{MN}$ that is measured at electrodes $A$ and $B$. The current density in the reciprocal array obeys the governing equation

$$\nabla \cdot \mathbf{j}_{MN} = \delta(\mathbf{r} - \mathbf{r}_M) - \delta(\mathbf{r} - \mathbf{r}_N), \quad (17)$$

where $\mathbf{j}_{MN} = \mathbf{j}_M - \mathbf{j}_N$ and $\mathbf{j}_M$ and $\mathbf{j}_N$ are normalized current densities corresponding to point sources at the positions $\mathbf{r}_M$ and $\mathbf{r}_N$ of electrodes $M$ and $N$.

We multiply Eq. (17) by $V_{AB}$ and integrate it over all space to give

$$\int V_{AB}\mathbf{j}_{MN} \cdot \hat{\mathbf{n}} dS - \int \mathbf{j}_{MN} \cdot \nabla V_{AB} dV = V_{AB}(\mathbf{r}_M) - V_{AB}(\mathbf{r}_N). \quad (18)$$

Here, $\hat{\mathbf{n}}$ is a unit outward normal, $dS$ and $dV$ indicate infinitesimal surface and volume elements, and $V_{AB}(\mathbf{r}_M)$ and $V_{AB}(\mathbf{r}_N)$ represent the potentials due to current injection at electrodes $A$ and $B$ measured at electrodes $M$ and $N$. We have used integration by parts and the divergence theorem on the left-hand side and a property of Dirac delta functions on the right. The first integral on the left-hand side is 0 because the normal component of the current density is zero on the ground surface and both the current density and the potential go to zero at infinite distance.

Using the definition of the apparent resistivity and Ohm's law, we can then see that

$$\rho_a = k_g \frac{V_{AB}(\mathbf{r}_M) - V_{AB}(\mathbf{r}_N)}{I_{AB}} = k_g \int \mathbf{j}_{MN} \cdot \mathbf{j}_{AB} \rho dV = \frac{k_g}{I_{AB} I_{MN}} \int \mathbf{J}_{MN} \cdot \mathbf{J}_{AB} \rho dV, \quad (19)$$

where $k_g$ is the geometrical factor for the array.

Eq. (19) tells us that the resistivity method is most sensitive to resistivity structure where the current densities due to the array and its reciprocal are strongly correlated. Areas of negative sensitivity correspond to regions where the current densities from the array and its reciprocal have components flowing in opposite directions. Note also that we could have multiplied Eq. (15) by $V_{MN}$ and followed the same procedure to get Eq. (19).

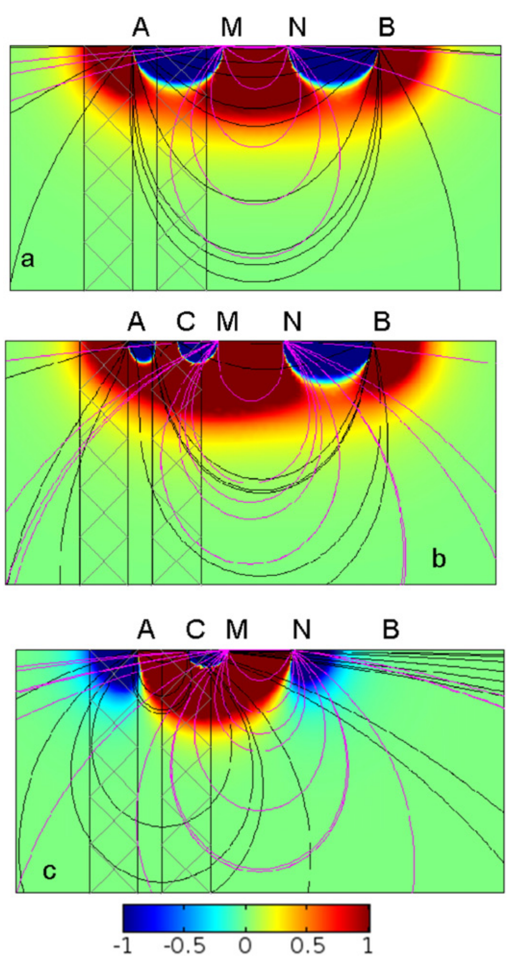


**Fig. 2.** Streamlines of the current due to the current electrodes are shown in black; those of the reciprocal array are shown in magenta. Color slice plots along the array line show the sensitivity function for ground of constant resistivity with current electrodes a) *A* and *B*, b) *A*, *B*, and *C* with $\alpha = 0.4$, and c) *C* and *A*. The color scale runs from −1 (blue) to 1 (red) and has been limited to this range. The gray cross-hatched regions are the locations of the dykes investigated in Section 5.

Barker (1979) obtained a formula that is mathematically equivalent to Eq. (19) for homogeneous ground starting with an electrostatic analogy. Park and Van (1991) have also shown that

$$\frac{\partial \rho_a}{\partial \rho_i} = k_g \int \mathbf{j_{MN}} \cdot \mathbf{j_{AB}} dV \quad (20)$$

for a piecewise constant resistivity distribution.

Fig. 2a shows a vertical slice through the electrode array of the sensitivity kernel calculated from Eq. (19) for the array shown in Fig. 1 when $\alpha = 0$ for homogeneous ground. Streamlines of $\mathbf{j}_{AB}$ are shown in black (flowing from *A* to *B*) while those of $\mathbf{j}_{MN}$ are shown in magenta (flowing from *M* to *N*). The color range has been saturated because the sensitivity goes to infinity at the positions of the electrodes. A similar signal contribution section was shown by Barker (1979).

### 2.3.1. Current cable connected to ground

When there is an extra current electrode *C* connected to electrode *A* that draws off current $\alpha I$, then the total normalized current density is $\mathbf{j}_{ABC} = (1-\alpha)\mathbf{j}_A + \alpha \mathbf{j}_C - \mathbf{j}_B$. The governing equation becomes

$$\nabla \cdot \mathbf{j}_{ABC} = [(1-\alpha)\delta(\mathbf{r}-\mathbf{r}_A) + \alpha\delta(\mathbf{r}-\mathbf{r}_C) - \delta(\mathbf{r}-\mathbf{r}_B)], \quad (21)$$

and the potential created is $V_{ABC}$.

Provided that there are no additional grounding points of the potential cables, the governing equation for the reciprocal array remains Eq. (17), and if we multiply it by $V_{ABC}$ and integrate the resulting equation over all space following the same procedure as above, we get the following equation for the apparent resistivity, which is in error because of the extra grounding point:

$$\rho_{ae} = \frac{k_g (V_{ABC}(\mathbf{r}_M) - V_{ABC}(\mathbf{r}_N))}{I_{ABC} I_{MN}} = k_g \int \mathbf{j}_{MN} \cdot \mathbf{j}_{ABC} \rho dV = \frac{k_g}{I_{ABC} I_{MN}} \int \mathbf{J}_{MN} \cdot \mathbf{J}_{ABC} \rho dV. \quad (22)$$

Here, $k_g$ is the geometrical factor for the array without electrode *C*. The difference between the correct apparent resistivity and the resistivity with electrode *C* can be calculated by subtracting Eq. (19) from Eq. (22) to get

$$\rho_{ae} - \rho_a = k_g \int \mathbf{j}_{MN} \cdot (\mathbf{j}_{ABC} - \mathbf{j}_{AB}) \rho dV = \alpha k_g \int \mathbf{j}_{MN} \cdot \mathbf{j}_{CA} \rho dV, \quad (23)$$

where $\mathbf{j}_{CA} = \mathbf{j}_C - \mathbf{j}_A$. Eq. (23) tells us that the error is largest when the current densities from the reciprocal array, the extra electrode, and the electrode to which it is connected are most strongly correlated. This agrees with the analysis of the homogeneous ground that the error is large when the extra current electrode is close to one of the potential electrodes. This is the case because the current density produced by the extra electrode and the current density that would be produced if injection was carried out at the potential electrode are strongly correlated if they are close together.

Fig. 2b shows streamlines of the current density for the array (black lines where current flows from *A* and *C* to *B*) and for the reciprocal array (magenta lines where current flows from *M* to *N*) for the array shown in Fig. 1 for ground with constant resistivity and $\alpha = 0.4$. We have used a large value of $\alpha$ so that the effects can be clearly seen in the figure. The sensitivity kernel as calculated from Eq. (22) is also shown in colors. Again, the color scale has been saturated because the sensitivity goes to infinity at the electrodes. As can be seen, the sensitivity kernel has been affected by the presence of the additional electrode at *C*.

Fig. 2c shows streamlines for the difference array connecting *A* and *C* (black lines with current flowing from *C* to *A*) as well as the streamlines for the reciprocal array and the sensitivity of the error as calculated from Eq. (23). As can be seen, the error is sensitive to the resistivity structure near electrodes *A*, *C*, *M*, and *N* but is insensitive to the resistivity near electrode *B*. The pattern of negative and positive sensitivity regions can be understood by the directions of the electrical current flows for the array connecting *A* and *C* and the one connecting *M* and *N*.

*2.3.2. Potential cable connected to ground*

If there is an extra potential electrode *C* connected to electrode *M* while current is injected at *A* and *B*, then Eq. (16) governs charge conservation whereas the reciprocal array has the governing equation

$$\nabla \cdot \mathbf{j}_{MNC} = (1-\alpha)\delta(\mathbf{r}-\mathbf{r}_M) + \alpha\delta(\mathbf{r}-\mathbf{r}_C) - \delta(\mathbf{r}-\mathbf{r}_N), \tag{24}$$

where the current density produces potential $V_{MNC}$. Multiplying Eq. (24) by $V_{AB}$ and integrating over all space and following the same procedure as above, the following expression for the apparent resistivity that is in error because of the extra potential electrode is derived

$$\rho_{ae} = \frac{k_g[(1-\alpha)V_{AB}(\mathbf{r}_M) + \alpha V_{AB}(\mathbf{r}_C) - V_{AB}(\mathbf{r}_N)]}{I_{AB}} = k_g \int \mathbf{j}_{MNC} \cdot \mathbf{j}_{AB} dV \cdot \tag{25}$$

We can again subtract Eq. (19) to get

$$\rho_a - \rho_{ae} = \alpha k_g \int \mathbf{j}_{CM} \cdot \mathbf{j}_{AB} dV, \tag{26}$$

where $\mathbf{j}_{CM} = \mathbf{j}_C - \mathbf{j}_M$. Note that Eqs. (23) and (26) are the same with the exchange of indices $A \leftrightarrow M$ and $B \leftrightarrow M$, as they must be by reciprocity.

Fig. 2a–c is equally applicable to the sensitivity of the error due to extra grounding of the potential electrode with the substitutions $A \leftrightarrow M$ and $B \leftrightarrow M$.

## 3. Comparison with field data

A small-scale linear dipole–dipole array was laid out with electrodes *A*, *B*, *M*, and *N* at positions 0 m, 2 m, 4 m, and 6 m. An additional electrode *C* was connected to *A* and moved along the array line. Usually, for a survey using four electrodes, *C* will be somewhere between the resistivity meter and *A*. In order to test our theory we have chosen to sample its effects all along the line. If the extra electrode is the result of a malfunctioning switch on a node in a multinode survey, the additional electrode may be at any of the node positions in the survey.

The survey was sufficiently small-scale that the ground was a reasonable approximation of an infinite half-space. Current was injected through *A* and *B* and potential measured between *M* and *N* giving apparent resistivity $\rho_{AB}^{MN}$, through *A* and *B* with *A* connected to *C* giving apparent resistivity $\rho_{ABC}^{MN}$, and through *B* and *C* only giving apparent resistivity $\rho_{BC}^{MN}$. All of the reciprocal pair combinations were also measured with injection at *M* and *N* for a total of six injections at each *C* position. Due to the linearity of the resistivity method, the parameter $\alpha$ can be calculated from

$$\alpha = \frac{\rho_{ABC}^{MN} - \rho_{AB}^{MN}}{\rho_{BC}^{MN} - \rho_{AB}^{MN}} = \frac{\rho_{MN}^{ABC} - \rho_{MN}^{AB}}{\rho_{MN}^{BC} - \rho_{MN}^{AB}}. \tag{27}$$

Here, the right-most expression corresponds to the apparent resistivities recorded from the reciprocal arrays. The values of $\alpha$ were found to vary from 0.35 to 0.7, and the values from the reciprocal arrays were in good agreement. The contact resistances of the electrodes were also measured directly using a FLUKE 1623 Earth/Ground tester and the values of $\alpha$ calculated using $\alpha = R_A / (R_A + R_C)$. The values calculated in this way were also in good agreement with the results of Eq. (27).

In Fig. 3a we plot $\delta\rho/\alpha$ against the position of the *C* electrode for the case of injection at *A* and *B* (solid line) and for the reciprocal array with injection at *M* and *N* (dashed line). As required by reciprocity, the curves are essentially the same, indicating also that unintentional grounding of the potential wire has an effect of the same magnitude as unintentional grounding of the current wire. Also shown is the prediction of Eq. (6) for an array with the same geometry, and it can also be seen to be in good agreement with the field measurements. We can clearly see the very large error that is caused when unintentional grounding of the current wire is close to the potential electrode and when an unintentional grounding of a potential wire is close to a current electrode.

Although it is difficult to see because of the scale of the graph, $\delta\rho/\alpha$ does not approach 0 as the position of *C* goes to infinity but instead approaches 1/2, which is the predicted value for this array from Eqs. (13) and (14).

In the vicinity of the negative peak at 4 m, the apparent resistivities $\rho_{ABC}^{MN}$, $\rho_{BC}^{MN}$, $\rho_{MN}^{ABC}$, and $\rho_{MN}^{BC}$ are all negative. Our resistivity meter returns the absolute value of the apparent resistivity, and so we had to change the sign of some of the measurements in post-processing.

In Fig. 3b, we plot $\delta\rho/\alpha$ against the position of the *C* electrode when the *C* electrode is connected to potential electrode *M* for a Wenner array with *A*, *B*, *M*, and *N* electrodes at fixed positions 0 m, 6 m, 2 m, and 4 m. Field measurements were made with voltage measured between *M* and *N* electrodes only, *M* and *N* with *M* connected to *C*, and between *C* and *N* only. The parameter $\alpha$ was calculated for each position using Eq. (27). The large peaks at the positions of the current electrodes can be clearly seen, and the measurements are in good agreement with the theory for an infinite half-space. As the *C* electrode gets far away, the relative error approaches $-1/2$, as predicted.

## 4. Results — constant resistivity ground

In Fig. 4a, we plot $\delta\rho/\alpha$ as a function of the position of the *C* electrode using Eq. (6) for a Schlumberger array when there is an additional grounding point of the cable connected to electrode *A*. The positions of the electrodes are indicated on the figure. The color range has been limited to $\pm 10$, but the error goes to infinity at the positions of the potential electrodes. The error is clearly very large when the additional grounding point is close to the potential electrodes. It can also be seen that there is a 0 contour of the error that passes through electrode *A* and that the error is fairly small if *C* is located near *B*. Also note that the error does not go to 0 as *C* is moved to infinity.

In Fig. 4b, we show a similar plot for the case of a pole–pole array, where *A* is the close electrode. As can be seen, the contours are now circular with center on the *M* electrode. If *A* was the infinite electrode, the contours would still be circles, but the values would be shifted with the position of the *C* electrode where it would not affect the measurement being at infinite distance. For all cases, the same results would be seen for a grounded potential cable connected to electrode *M* if we exchanged $A \leftrightarrow M$ and $B \leftrightarrow N$. Plots for other array geometries can be easily generated by evaluating Eqs. (6) and (11).

## 5. Effects of variable resistivity

We investigate the effects on the error caused by a leakage current in ground with variable resistivity for the array geometry used in Fig. 1, where the resistivity is varied in vertical dykes of width 0.3. The vertical dykes are shown as the gray cross-hatched regions in Fig. 2. As can be seen from Fig. 2c, the sensitivity is dominantly negative for the error for the dyke located to the left and is dominantly positive for the dyke on the right. In the calculations to follow, the resistivity is constant outside of the dykes with value $\rho_1 = 1\ \Omega$ m and $\alpha = 0.4$.

Fig. 5a and b shows the results of varying the resistivities within the right and left dykes. The variable resistivity within the dyke is represented by $\rho_2$. Circles show the results of numerical simulations using the finite element modeling package, Comsol (Butler and Sinha, 2012), while the solid lines are the predicted errors calculated from

$$\delta\rho = \alpha k_g \left( \rho_1 \int \mathbf{j}_{MN} \cdot \mathbf{j}_{CA} dV_1 + \rho_2 \int \mathbf{j}_{MN} \cdot \mathbf{j}_{CA} dV_2 + \rho_1 \int \mathbf{j}_{MN} \cdot \mathbf{j}_{CA} dV_3 \right). \tag{28}$$

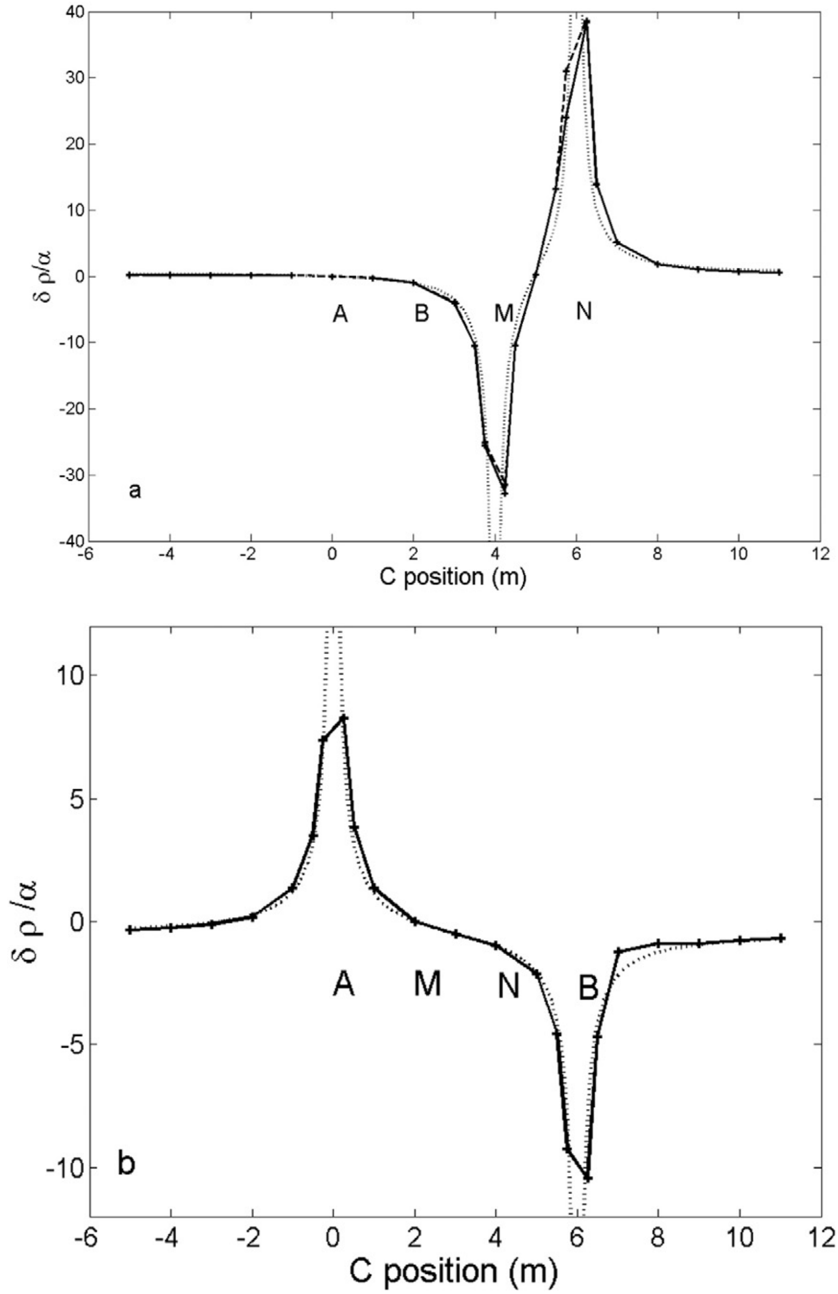


**Fig. 3.** a) Relative error divided by $\alpha$ for an array with extra grounding of the current electrodes (dashed line), extra grounding of the potential electrodes (solid line), and the theoretical relative error for an infinite half-space (dotted line) for a dipole–dipole array. Horizontal positions of the electrodes are indicated. Electrode *C* is connected to electrode *A*. b) Relative error divided by $\alpha$ for the case of an additional potential electrode *C* connected to *M*. The solid line represents the field data while the dotted line shows the theoretical error. The positions of the electrodes in the Wenner array are indicated.

In Eq. (28), volumes 1, 2, and 3 indicate the simulation volumes to the left of the dyke, within the dyke, and to the right of the dyke, and the current densities used in the integrals are those for an infinite half-space of constant resistivity, or equivalently, when $\rho_2 = \rho_1$.

As predicted, for the rightmost dyke, the error increases with increasing resistivity in the dyke. However, the rate of increase decreases as the dyke becomes increasingly resistive. This occurs because currents become concentrated in conductive regions and avoid resistive regions and so the function $\mathbf{j}_{MN} \cdot \mathbf{j}_{CA}$ decreases as $\rho$ increases in the dyke.

For the leftmost dyke, the error decreases with increasing resistivity because the currents from the forward and reciprocal arrays flow in opposite directions. Again, the rate of decrease decreases with increasing resistivity.

The range of the error induced by the extra electrode is greater for the rightmost dyke than the one to the left of *A* because of the larger magnitude of the sensitivity in that region. For both cases, evaluating the integrals in Eq. (28) using the current densities appropriate for ground with constant resistivity, which corresponds to a linear approximation, does a reasonable job of predicting the actual variation in the error only for a range of resistivity within roughly a factor of 3 or 4.

## 6. Discussion

In practice, the location and contact resistance of an unintentional grounding point are generally not known. If a multinode 2D or 3D survey has been carried out where the cables are not moved during a

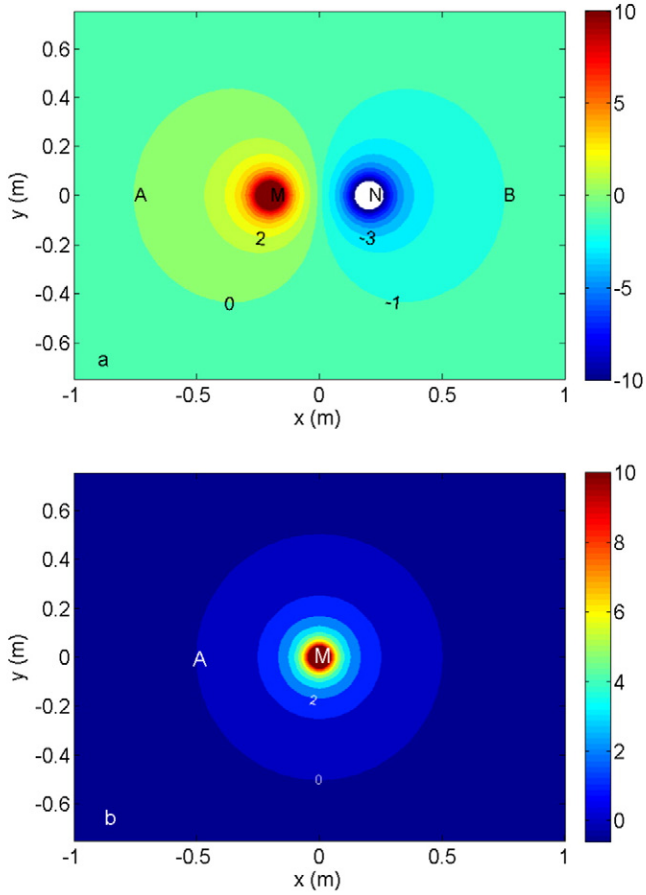


**Fig. 4.** $\delta\rho/\alpha$ for a) a Schlumberger array and b) a pole–pole array, where the additional grounding point is connected to electrode *A*.

survey, the unintentional grounding may have been the same for all current injections and measurements, and it may be possible to theoretically determine the location and $\alpha$ of the unintentional grounding using Eqs. (6), (11), (23), and (26). In particular, very large positive or negative apparent resistivities will be recorded when a current electrode in an array is near an unintentional potential electrode or when a potential electrode is near an unintentional current electrode. It might be possible to find more subtle unintentional grounding points by including the positions and contact resistances of such points as additional parameters in an inversion.

When designing a survey, one can estimate the maximum error that could be caused by an unintentional grounding point. If a maximum ratio of contact resistance of the intentional to the unintentional electrode is assumed (Zohdy (1968) estimated it to be 0.01) then Eqs. (6) and (11) can be used, where $r_{CM}$ and $r_{CN}$ are taken to be the closest approaches of the current electrode cables to the potential electrodes and $r_{AC}$ and $r_{BC}$ are taken to be the closest approaches of the potential electrode cables to the current electrodes. It may be possible to keep current/potential cables sufficiently far away from potential/current electrodes to minimize the error, or special care may be taken to insure that cables that are located close to electrodes are well insulated. However, it should be kept in mind that Eqs. (23) and (26) indicate that if there is a significant variation in resistivity in regions where an array made of the unintentional array and the electrode to which it is connected are very sensitive, then the error could deviate significantly from that predicted for a homogeneous infinite half-space.

Measurements with very large errors, caused by unintentional groundings of current/potential cables near potential/current electrodes, are likely to be noted because the apparent resistivity will be

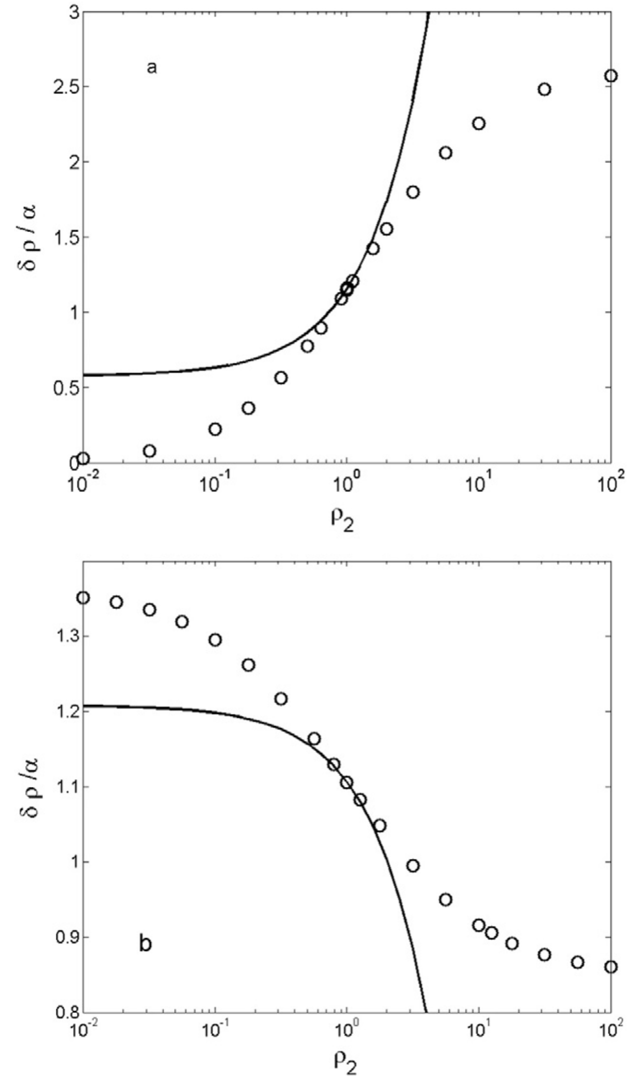


**Fig. 5.** The relative error in the apparent resistivity as a function of the resistivity in a 2D dyke of width 0.3 (circles). The solid line represents the estimated error predicted using the sensitivities of the various regions when $\rho_2 = 1$. The value of $\alpha$ used was 0.4. a) The 2D dyke is centered on the *C* electrode b) the 2D dyke is just to the left of electrode *A*. Positions of the dykes are shown in Fig. 2.

outside of the expected range. However, more subtle, but still significant, errors may occur if the distance from the unintentional current/potential electrode is similar to the typical array spacing to the closest potential/current electrode. In order to detect such an error for the case of current leakage, each of the current electrodes in turn could be disconnected at the electrode, during a current injection pulse. If a significant current flowed while one of the electrodes was disconnected, this would indicate a point of current leakage somewhere along the current carrying wire as Zohdy (1968) suggested. In a multinode system, such a test could be programmed into the injection sequence. Alternatively, current meters could be located on each of the current electrodes and a difference in the current at the two electrodes would be indicative of current leakage. If current was measured at the two current electrodes, the parameter $\alpha$ would be known and a better estimate of the upper bound of the error in the apparent resistivity could be calculated.

The detection of an unintentionally grounded potential wire could be carried out by placing the two intended potential electrodes very close together in the ground, but not touching. If during current injection at two other electrodes, there was still a significant voltage difference between the potential electrodes, then an additional grounding point would be suspected. This method would clearly be very laborious and would involve moving at least one of the potential wires, potentially altering its contact with the ground and hence the presence of an error. Alternatively, an additional potential wire could be used, along with an additional voltmeter, to measure the potential between the intended electrode and the original resistivity meter. If during current injection at two other electrodes, this potential was substantially greater than 0, then an unintentional potential grounding would be inferred to be occurring somewhere along the potential cable. Again, this would be laborious for a four electrode survey and it would be impractical to double all of the cables in a multinode survey. A better way to detect an unintentional grounding on the potential wire would be to inject current in the potential electrodes during a test and use one of the tests described above to determine if a current leakage was taking place.

## 7. Conclusions

We have derived formulas for the magnitude of the error in the measured apparent resistivity for general four-electrode arrays in homogeneous ground when there is an unintentional grounding of one of the current or potential electrodes. The magnitude of the error is proportional to the apparent resistivity that would be measured between the potential electrodes caused by current injection at the leakage point and the electrode to which it is attached for the case of leakage current. The magnitude of the error is proportional to the apparent resistivity that would be measured between the extra grounding point and the electrode to which it is attached that is caused by current injection for the case of a grounded potential cable. These formulas have been compared with the results of small-scale dipole–dipole and Wenner arrays, and they have been shown to be in excellent agreement.

Formulas for the sensitivity of the error to resistivity variations in the ground have also been derived. The sensitivities are proportional to those for arrays made up of the additional array and the electrode to which it is connected. The sensitivity of the error to resistivity variations can be both negative and positive.

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